$$\log x_j \begin{vmatrix} 2 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 2 & 1 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 2 & 1 \\ 1 & 0 & 0 & \cdots & 0 & 2 \end{vmatrix} = \begin{vmatrix} 2 & 1 & 0 & \cdots & \log a_1 & 0 & \cdots & 0 \\ 0 & 2 & 1 & \cdots & \log a_2 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & \log a_j & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & 0 & 0 & \cdots & \log a_n & 0 & \cdots & 2 \end{vmatrix}.$$

Denoting the determinant in the left member of the last equation by D and the one in the right member by A_j, we have

$$\log x_j = \frac{A_j}{D},$$

or

$$x_j = e^{\frac{A_j}{D}} \; (j = 1, 2, \cdots, n).$$

Also solved by the PROPOSER, who used logarithms as in the last solution, though he did not use determinants.

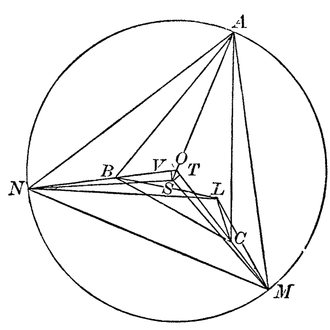
GEOMETRY.

424. Proposed by H. E. TREFETHEN, Colby College.

In a given triangle ABC, determine by geometric demonstration the point O such that the sum of the distances, $AO + BO + CO$, shall be a minimum.

SOLUTION BY M. E. GRABER, Heidelberg University.

On each of the three sides of the triangle ABC, describe a segment of a circle to contain an angle of 120°. The arcs of these circles intersect at a point O, about which the angles are each 120°. Then $AO + BO + CO$ is a minimum. Suppose $OA > OB$ or OC. Using OA as a radius describe a circumference. Produce OC and OB to meet the circumference at M and N. Let L be any other point in ABC.



Draw LC, LM, LB, LN, AM, MN, NA. Then $AM = MN = NA$ and $LA + LN + LM > OA + ON + OM$. In order to prove this relation, it is evidently sufficient to prove that $OA + ON + OM <$ the sum of the distances from S (the vertex of an isosceles Δ on MN with the same altitude as ΔLMN)

to A, M, and N. With MS and NS as radii describe arcs cutting MO and NO in T and V. Join T and S, V and S, by straight lines. Then $\angle OVS$ is obtuse and therefore $\angle OSV$ is $< 30°$. When $\angle SOV = 60°$ and $\angle OSV = 30°$, $OV = \frac{1}{2}OS$. When $\angle OSV < 30°$, $OV < \frac{1}{2}OS$ and $OV + OT < OS$. Hence $SM + SN + SA > AO + OM + ON$ and $OM + ON + OA < LM + NL + LA$. Since $LC + CM > LM$ and $LB + BN > LN$, we have $LC + CM + LB + BN + LA > OA + OB + BN + OC + CM$. Hence, $LA + LB + LC > OA + OB + OC$.

Also solved by C. N. Schmall, David L. MacKay, A. M. Harding and Harvy Roeser.

CALCULUS.

336. Proposed by EVA S. MAGLOTT, Ada, Ohio.

If a right cone stands on an ellipse, prove that its superficial area is $\frac{\pi}{2}(OA + OA')(OA \cdot OA')^{\frac{1}{2}}$ times $\sin\alpha$, where O is the vertex of the cone, A and A' extremities of the major axis of the ellipse, and α is the semi-angle of the cone.

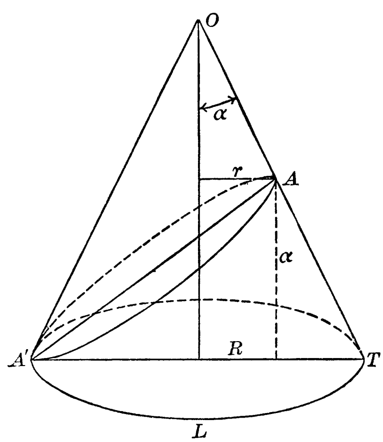
Solution by Elmer Schuyler, Brooklyn, N. Y.

The formula for the surface $AA'LT$ is

$$\frac{\pi}{R-r}\sqrt{a^2 + (R-r)^2}\,\{R^2 - \tfrac{1}{2}(R+r)\sqrt{Rr}\}.$$

(See Finkel's *Mathematical Solution Book*, pp. 318–319).

Here $r = OA\sin\alpha$, $R = OA'\sin\alpha$, and $a = (OA' - OA)\cos\alpha$.



For brevity, put $OA = K$, $OA' = K'$. Then this formula becomes, after slight reduction,

$$\frac{\pi}{\sin\alpha}\cdot\sin^2\alpha\{K'^2 - \tfrac{1}{2}(K+K')\sqrt{KK'}\} = \frac{\pi}{2}\sin\alpha\,\{2K'^2 - (K+K')\sqrt{KK'}\}.$$